

## Chapter 10

10.1

a)

	X	
Y	0	16
0	.25	.25   0.5
8	.25	0   0.25
16	0	.25   0.25
	0.5	0.5

b)

$$E[X] = 0 \times 0.5 + 16 \times 0.5 = 8$$

$$E[Y] = 0 \times 0.5 + 8 \times 0.25 + 16 \times 0.25 = 6$$

$$E[XY] = 0.25 \times 0 \times 0 + 0.25 \times 16 \times 0 + 0.25 \times 8 \times 0 + 0 \times 8 \times 16 + 0 \times 0 \times 16 + .25 \times 16 \times 16 = 64$$

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 64 - 8 \times 6 = 16$$

c)

$$\text{Var}(X) = .5 \times (0 - 8)^2 + 0.5 \times (16 - 8)^2 = 64$$

$$\text{Var}(Y) = 0.5 \times (0 - 6)^2 + 0.25 \times (8 - 6)^2 + 0.25 \times (16 - 6)^2 = 44$$

$$\text{Correlation coefficient} = \text{Cov}(X, Y) / \sqrt{\text{Var}(X)\text{Var}(Y)} = 16 / \sqrt{64 \times 44} = 16 / 53.07 = 0.302$$

10.5

a)

	a		
b	0	1	2
0	8/72	6/72	10/72   1/3
1	12/72	9/72	15/72   1/2
2	4/72	3/72	5/72   1/6
	1/3	1/4	30/72

b)

$$E[X] = 1/3 \times 0 + 1/4 \times 1 + 30/72 \times 2 = 1.08$$

$$E[Y] = 1/3 \times 0 + 1/2 \times 1 + 1/6 \times 2 = 5/6$$

Ignoring the data from the first row and the first column, where we are multiplying by 0's,

$$E[XY] = 1 \times 1 \times 9/72 + 1 \times 2 \times 15/72 + 2 \times 1 \times 4/72 + 2 \times 2 \times 5/72 = .93$$

$$\text{Cov}(X, Y) = .93 - 0.89 = 0.04$$

10.8

a)

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

$$\text{So } E[X^2] = \text{Var}(X) + (E[X])^2 = 4 + 4 = 8$$

b)

$$E[-2X^2 + Y] = E[Y] - 2E[X^2] = 3 - 2 \cdot 8 = -13$$

10.11

$$\text{Cov}(rX + s, tY + u) = rt\text{Cov}(X, Y)$$

$$T = 9/5X + 32$$

$$S = 9/5X + 32$$

$$\text{So } \text{Cov}(T, S) = 9/5 \times 9/5 \times \text{Cov}(X, Y) = 3 \times 81/25 = 9.72$$

Since the correlation coefficient does not change with changing units, it is still 0.8